

Introduction on the Magnetosphere Shielding Model - MSM

To reach a location within the Earth's magnetosphere, charged solar energetic particles have to penetrate through the shielding offered by the magnetic fields around the Earth. The rigidity, R defined as momentum over unit charge, is often used in characterization of the interaction of charged particles with the geomagnetic field.

The magnetosphere consists of the internal geomagnetic field which is generated by the motion of molten iron alloys in the Earth's outer core and the external magnetospheric fields which are due to variability of the solar wind and many others. For the internal field, every five years, the International Association of Geomagnetism and Aeronomy (IAGA) issues a new set of Gauss coefficients defining the new IGRF model [1]. The MSM used IGRF11 was released in 2010. For the external field, the semi-empirical Tsyganenko magnetic field models have been widely utilized in the space physics community for many years. They are the best-fit representations for the magnetospheric field, based on a large number of satellite observations. There are currently four main versions, and MSM used the 1989 version (Tsy89) [2] which was primarily developed as a simple empirical approximation for the global magnetosphere, binned into several intervals of the disturbance index K_p .

For accurate calculation of the rigidity, it requires the use of cosmic ray tracing method in conjunction with the detailed field models, i.e. the use of the IGRF and Tsy89. Today the most commonly used code for this is MAGNETOCOSMICS [3], based on the Geant4 Toolkit [4]. It was developed by University of Bern with ESA support,

Rigidity calculation using the particle trajectory tracing method is very computation intensive and one track could take up to 1 minute of CPU time even on today's fast desktop machines. So instead of calculating the cut-off on-the-fly, interpolations based on pre-calculated vertical cut-off maps are generally used.

Tsy89 external field model

We decided to use the Tsy89 model as the outfield model because it only employs a single parameter l_{opt} to describe the status of the field. The use of a single active level parameter for the external field greatly simplifies the parameterizations of the magnetosphere. However as the original Tsy89 model only applies to K_p index up to 6, it is necessary to use the Boberg extension [5] to cover the more disturbed conditions with $k_p > 6$. However following discussions with Smart & Shea [6], we decided to use a Dst step size of -50 nT instead of the original Boberg step size of -100 nT. The new extension scheme is :

Kp	Extended TS89 Model	
	lopt	Dst (nT)
0, 0+	1	n/a
1-, 1, 1+	2	n/a
2-, 2, 2+	3	n/a
3-, 3, 3+	4	n/a
4-, 4, 4+	5	n/a
5-, 5, 5+	6	n/a

6-,6, 6+	7	n/a
7-,7,7+	7	-50
8-,8,8+	7	-100
9-,9	7	-150

The pre-calculated cut-off maps

The pre-calculated vertical rigidity cut-off maps were generated over the following set of parameters

- Altitude: 450 km.
- Epoch: 1955 to 2015, in 5 year step
- at 0, 3, 6, 9, 12, 15, 18, and 21 UT
- Geomagnetic indices: Kp =0, 1, 2, 3, 4, 5, 6, 7, 8, and 9

A set of 1040 global rigidity cut-off maps with 5° x 5° resolution were produced. It is also necessary to have the corresponding McIlwain "L" parameter and the magnetic latitude included for each of the map data points. The IRBEM library [7] was used for calculating both.

Interpolation algorithms

It has been found [8] [9][10] that the cut-off rigidity change with radial distance is proportional to the inverse power of the McIlwain "L" parameter, i.e. L^{-2} . The "L" parameter can be utilized in the cut-off equation for the cosine squared of the magnetic latitude; hence the vertical cut-off rigidity equation used has the form of:

$$R_{vc} = \frac{V(k)}{L^2} \quad (1)$$

where $V(k)$ is the equivalent of the Störmer constant for the vertical cut-off rigidity. There is no constant global value for $V(k)$, due to the displacement of the effective magnetic centre from the geo-centre and the deviation of the IGRF magnetic field model from a dipole, there is a slightly different $V(k)$ at each location.

These characteristics of the "L" parameter form the basis of our basic cut-off rigidity interpolation procedure. With the pre-calculated maps, the cut-off rigidity at a given time and location in the magnetosphere can be determined by interpolation. The pre-calculated map files provide the cut-off values and the "L" value, and with these the constant $V(k)$ at every grid point can be obtained.

For any location at the same altitude of the maps, i.e. 450 km, we determine the cut-off value at the spacecraft latitude and longitude in the following manner:

1. From the given date determining which two sets of maps are used, e.g., at epochs 2000 and 2005.
2. Determine the Kp value and Universal Time for which the interpolation is to be made.
3. For each set of the maps, do the following
4. Calculate the $V(k)$ value for each grid point, using equation(1).
5. Use the geographic coordinates of the location of the spacecraft locate the appropriate "box" for which the cut-off rigidity value is desired.

6. Interpolate in latitude on each side of the box using a weighted average of the change in the "L" coordinate from the top to the bottom of the box. This establishes the constants $V(k)$ for the cut-off rigidity equation(1) for the desired latitude at the "left" and "right" sides of the box.
7. Employ linear interpolation to determine $V(k)$ at the longitude of the spacecraft.
8. Calculate the "L" value for the geographic coordinates of this location.
9. Determine the vertical cut-off rigidity for this location (at 450 km altitude) using the vertical cut-off rigidity equation (1)
10. Employ linear interpolation between the two vertical cut-off rigidities to determine the value for the specified date.

If the spacecraft altitude is not at 450 km, then the McIlwain "L" coordinate is computed at the spacecraft position and altitude and "L" interpolation is used to extrapolate from the cut-off value at 450 km altitude to the spacecraft altitude. The previously computed values of $V(k)$ for the spacecraft latitude and longitude are used with the "L" coordinate of the spacecraft latitude, longitude and altitude to compute the cut off value.

Cut-off at other directions

The interpolated vertical geomagnetic cut-off rigidity can be transformed to any given direction using the Störmer theory, i.e., the cut-off rigidity, R_c , for any given direction (ε, ϕ) can be derived from the vertical cut-off rigidity, R_{vc} :

$$R_c = \frac{4R_{vc}}{(1 + \sqrt{1 - \sin \varepsilon \sin \phi \cos^3 \lambda_m})^2}$$

Where λ_m is the magnetic latitude of the given location.

Strictly speaking this formula is only applicable to dipole magnetic field, however it is a good enough approximation for near Earth orbits.

The average transmission function

For charged particles with a rigidity of R arriving from interplanetary space at the direction of (ε, ϕ) , its transmission function is defined as unity if $R > R_c$ and zero otherwise:

$$\begin{aligned} T(\varepsilon, \phi, R) &= 1; R > R_c \\ T(\varepsilon, \phi, R) &= 0; R \leq R_c \end{aligned}$$

However the more commonly used parameter for isotropic radiation environment is the averaged transmission function, which is simply the average transmission function over the 4π solid angles:

$$T_{aver}(R) = \iint T(\varepsilon, \phi, R) \sin \varepsilon \, d\varepsilon d\phi / 4\pi$$

Earth shadowing

The Earth shadowing effect, $SE(h)$ is calculated using the simple geometric formula:

$$SE(h) = 2\pi \times \left(1 + \frac{\sqrt{(r_e + h)^2 - r_e^2}}{r_e + h}\right)$$

Where h is the altitude and r_e is the radius of the Earth.

Running the orbit tool and MSM tool

Users will specify the orbit parameters through an interface similar to the one used in SPENVIS. The orbit parameters are extracted into a namelist file, which is stored in the database together with the orbit file.

When running the MSM tool, the SEPTEM system selects a previously generated trajectory file and a single value for K_p . The MSM tool evaluates the transmission factor as a function of rigidity for every location and produces an orbit averaged transmission “spectrum”.

Application of the cut-off “spectrum”

There are three ways in which the MSM outputs could be used to evaluate magnetic shielding in the context of SEPTEM:

1. Fold the transmission spectrum with an interplanetary spectrum obtained with the statistical tools.
2. Apply the cut-offs on the fly when building a statistical model, using an orbit averaged transmission spectrum and a weighted average of K_p over each event.
3. Similar to 2, but use a Monte Carlo type simulation of varying the phase of the orbit over each event and using instantaneous values of K_p .

Option 1: folding interplanetary spectra

This is the baseline solution which corresponds to solutions implemented by SPENVIS, for instance.

Option 2: run a statistical model on statically shielded event spectra

In this approach, the statistical tools are run on an event list where the particle flux is attenuated using the MSM. For each event, the historic time record of K_p is weighted using, for instance, the relative instantaneous flux, resulting in a single K_p value per event. This K_p value is used for each event to drive the MSM tool to obtain orbit averaged transmission coefficients as a function of rigidity. The rigidity defined by particle type and energy channel is used to interpolate in rigidity to obtain an average transmission factor for the event. The scaled fluence or flux is then used instead of the original values in the statistics.

In order to optimise the calculation, the MSM tool can be run at the start for each of the K_p values used in the Tsy89 model, so that a list of transmission factors as a function of K_p is available for interpolation.

Implementing this method implies that the historical time record of K_p needs to be added to the database.

Option 3: run a statistical model on dynamically shielded event spectra

This method is similar to method 2, with the difference that the variation of K_p during the events is taken into account. Instead of using a single, weighted average value of K_p per event, the MSM is run with the time series of K_p , which means that the MSM tool needs to be extended to handle a time series input of K_p . The time profile of the event flux is then folded with the instantaneous transmission function. The resulting event fluence or peak flux (after transmission) is then used by the statistical tool.

An additional complication is that the time variation of the cut-off over the orbit depends on the phasing of the trajectory. Therefore, the calculation should be repeated (for each event) a number of times whereby the start of the trajectory is generated by a Monte Carlo method.

References

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